

Overview: Sides of the Unitarity Triangle

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This is an opening talk for the workshop and is intended to be provocative. We take a stab at the following questions: How well do we really know the sides of the unitarity triangle? What else could we do to improve? (I propose several new measurements). What precision should we ultimately aim at in the determination of CKM elements? What have we learned so far about flavor physics? Where do we go from here?

I. INTRODUCTION

How precise should we ultimately measure the elements of the CKM[1, 2] matrix? I am not asking what is the ultimate precision afforded by present day methods, but rather, how precisely do we need to know them. A rather common answer is that one should aspire to determine them as well as possible given available methods because the CKM elements are fundamental constants of nature, as fundamental as any other coupling in the Lagrangian of the Standard Model of electroweak and strong interactions (SM). But I find this answer lame and naïve, particularly when the effort is rather expensive both in real money and in human capital. A much better answer is obtained by estimating realistically how large deviation due to new physics could be.

It is not difficult to find extensions of the standard model that would give deviations from expected measurements just beyond the precision attained to date. For example, one can take the minimal extension to the supersymmetrized SM (the MSSM), and choose parameters appropriately, that is, on the verge of being ruled out (or discovered). But this is contrived, and not a reasonable way to answer our question.

I consider here two ways of estimating the precision with which we need to determine CKM elements. The first one consists of verifying that the CKM matrix is unitary. The second one asks the question, what precision is needed to exclude new physics at the TeV scale? These questions will be explored below in Secs. II and III, respectively. I have dismissed a third possibility, which asks what precision is needed to limit models of flavor. By these I mean models of new physics that attempt to predict the numerical entries in the CKM matrix. The reason I dismiss this question is the following. Assume the “landscape” of CKM matrices produced by models is discrete; else arbitrary precision will not distinguish between models. Then the precision needed to distinguish models is the typical separation between points on the landscape. However, I don’t see today any limitation for theorists to invent new models that are less coarse in their distribution of points on the landscape.

I will also discuss the current precision of some of the

measurements of sides of the unitarity triangle. I will try to be aggressively critical of the estimated errors, to keep us all honest. This is discussed in Sec. IV. In so doing I will propose some measurements that could be done in the next round of experiments. My conclusions, really wild speculations, are found in Sec. V.

II. UNITARITY

A measure of the needed precision for CKM elements can be estimated from testing whether the CKM matrix is unitary, as it should. The CKM matrix would fail to be unitary if there existed a fourth generation of quarks. Now, we all know that this possibility is nearly excluded. For one thing, one would expect an additional generation of leptons as well and a fourth light neutrino is excluded by the precise measurement of the width of the neutral Z vector boson and the global electroweak fit. Nevertheless I believe this is a useful test, and in any case creative theorists have invented models surmounting these difficulties; see, for example, Ref. [3].

Present data already gives a check that the matrix is approximately unitary. The sum of the square-modulus of the entries of a row (or column) of a unitary matrix equals unity. The best determined sums are[4]

$$\sum_{i=d,s,b} |V_{ui}|^2 = 0.9992 \pm 0.0011 \quad (1)$$

$$\sum_{i=u,c,t} |V_{id}|^2 = 1.001 \pm 0.005 \quad (2)$$

$$\sum_{i=u,c,j=d,s,b} |V_{ij}|^2 = 2.002 \pm 0.027 \quad (3)$$

so the first row and column are unitary to 1.1 and 5 per mil, respectively, and subtracting (1) from (3) the second row is to 3%. What kind of deviations from unity could (or should) we expect?

To answer this we need a guess. The Wolfenstein parametrization, which you can find many times over in this volume, indicates the texture of the CKM matrix,

$$V_{\text{CKM}}^{(3)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (4)$$

where $\lambda \sim 0.22$ is a small parameter. I propose two guesses that extend this texture to the case of a 4×4

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matrix in ways that seem natural. The first is an attempt at pattern matching, consistent with unitarity:

$$V_{\text{CKM}}^{(4)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^5 \\ \lambda & 1 & \lambda^2 & \lambda^4 \\ \lambda^3 & \lambda^2 & 1 & \lambda^2 \\ \lambda^5 & \lambda^4 & \lambda^2 & 1 \end{pmatrix} \quad (5)$$

This gives a sobering estimate for the expected deviation from unity in the three rows of the CKM:

$$\begin{aligned} 1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) &\sim \lambda^{10} \sim 3 \times 10^{-7} \\ 1 - (|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2) &\sim \lambda^8 \sim 5 \times 10^{-6} \\ 1 - (|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2) &\sim \lambda^4 \sim 2 \times 10^{-3} \end{aligned}$$

Present tests are far from sensitive to these guess dependent deviations. The best bet to find these is to look in the third row (with obvious caveats about high precision measurements in the first two rows).

The second guess is an attempt to be least conservative, that is, take the fourth row and column as large as possible given the texture in (4), and 4×4 unitarity. Taking also into account the fact that V_{tb} is poorly known ($|V_{tb}| = 0.77^{+0.18}_{-0.24}$ [4]), we guess

$$V_{\text{CKM}}^{(4)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^3 \\ \lambda & 1 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 & \lambda \\ \lambda^3 & \lambda^2 & \lambda & 1 \end{pmatrix} \quad (6)$$

The lower-right 2×2 block involves a mixing angle much like the Cabibbo angle in the upper-left 2×2 block. In this case the required precision in the first and second row is not far from what is accomplished to date, $\lambda^6 \sim 1 \times 10^{-4}$ and $\lambda^4 \sim 2 \times 10^{-3}$, respectively. And deviations in row/column three could be large, which suggests looking for unitarity violations in the third row or column. Surprisingly little thought goes into the question of how to determine the third row without using unitarity of the 3×3 matrix. There are many simple analysis waiting to be done. For example, consider the indirect measurement of three parameters a , b and c given by

$$a = |V_{td}V_{ts}|, \quad b = |V_{td}V_{tb}|, \quad c = |V_{ts}V_{tb}| \quad (7)$$

These three parameters could be obtained through a combination of measurements of mixing and/or decays of neutral strange and bottom mesons. From these three measured quantities one extracts the third row:

$$|V_{td}| = \frac{ab}{c}, \quad |V_{ts}| = \frac{ac}{b}, \quad |V_{tb}| = \frac{bc}{a} \quad (8)$$

One should be mindful that this determination is based on the virtual effect of the top quark and if a fourth family were present it would contribute to the three parameters, so the ratios (8) would fail to give the values of the third row CKM elements. Still, this would give an independent test of unitarity of the third row, so this seems worth pursuing.

III. TEV PHYSICS

In the absence of new dynamics radiative corrections would render the mass scale of the electroweak theory comparable to the Planck scale. New physics at the TeV scale is generally invoked to explain this “hierarchy problem.” But quark mass terms break the electroweak symmetry group, so the quark mass matrices are necessarily connected to this new physics. New “higgs dynamics” at the TeV scale must incorporate new flavor physics too.

This suggests another criterion for the required precision in the determination of CKMs, namely, enough that we can see clearly the effects of this new flavor physics originating from the new, TeV-scale dynamics. To describe the effects of new TeV dynamics at below TeV energies one simply extends the Lagrangian of the SM by operators of dimension higher than four, suppressed by powers of the new physics scale, Λ . The work in [5, 6] lists all operators of dimension five and six and analyzes some of their effects. Ignoring operators mediating flavor changing neutral currents (FCNC), $\Lambda \sim$ a few TeV is consistent with experiment. But if the coefficient of FCNC operators is given by dimensional analysis, then $\Lambda \sim$ a few TeV is strongly excluded. A much larger scale, $\Lambda \sim 10^4$ TeV, is still consistent with experiment, but then a hierarchy problem reappears.

Let \mathcal{A} denote the amplitude for some process which we write as the sum of SM and new physics pieces, $\mathcal{A} = \mathcal{A}_{\text{SM}} + \mathcal{A}_{\text{New}}$. If this proceeds at tree level in the SM we estimate, roughly,

$$\mathcal{A}_{\text{SM}} \sim \frac{g^2}{M_W^2} \times \text{CKM} \quad \text{and} \quad \mathcal{A}_{\text{New}} \sim \frac{1}{\Lambda^2}, \quad (9)$$

where the factor “CKM” stands for some combination of CKM elements. If we want to be sensitive to the second term the uncertainty in the first one should be no larger than the expected size of new physics effects:

$$\frac{\delta(\text{CKM})}{\text{CKM}} \sim \frac{1}{\text{CKM}} \frac{1/\Lambda^2}{g^2/M_W^2} \sim 1\% \times \left(\frac{0.03}{\text{CKM}} \right) \left(\frac{10 \text{ TeV}}{\Lambda} \right)^2 \quad (10)$$

Repeat now the power counting leading to (10), but for processes involving FCNC. These require at least one loop in the SM, but not in the new physics. We now estimate

$$\mathcal{A}_{\text{SM}} \sim \frac{\alpha}{4\pi \sin^2 \theta_w} \frac{g^2}{M_W^2} \times \text{CKM}, \quad (11)$$

so that

$$\begin{aligned} \frac{\delta(\text{CKM})}{\text{CKM}} &\sim \frac{1}{\text{CKM}} \frac{1/\Lambda^2}{(\alpha/4\pi \sin^2 \theta_w)(g^2/M_W^2)} \\ &\sim 400\% \times \left(\frac{0.03}{\text{CKM}} \right) \left(\frac{10 \text{ TeV}}{\Lambda} \right)^2 \end{aligned} \quad (12)$$

This is an underestimate since for SM’s FCNC the CKM combination is smaller than 0.03.

One can measure the CKM elements from processes that are tree level in the SM with little contamination from new physics, and then use those values to compute FCNC's in the SM to look for new physics. Since the CKMs are certainly measured to better than 30%, let alone 400%, either there is no solution to the hierarchy problem or there is some mechanism that automatically reduces the FCNCs of the new physics. In the absence of this automatic mechanism we have no basis for estimating the required precision in the CKM determination: it is determined by the scale Λ of which we know nothing. But this is not the case if we understand what this automatic mechanism is. More on this later, in Sec. V.

IV. SIDES DETERMINATION

Let us pause to look at the status of the determination of the CKM elements. This whole workshop is a huge study of this question. I pick here three elements for critical study, as I was asked to do by the organizers of the workshop.

a. $|V_{td}/V_{ts}|$ The magnitudes of V_{td} and V_{ts} are determined from measurements of neutral B_d and B_s oscillations, respectively. The big news this year is the precise measurement of the B_s mixing rate at Tevatron experiments[7, 8]. While $|V_{ts}|$ does not provide direct information on the apex of the unitarity triangle, the ratio $|V_{td}/V_{ts}|$ does. The interest in the ratio stems from the cancellation of hadronic uncertainties:

$$\frac{|V_{td}|}{|V_{ts}|} = \xi \sqrt{\frac{\Delta m_s m_{B_s}}{\Delta m_d m_{B_d}}}, \quad \text{where} \quad \xi^2 \equiv \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2}. \quad (13)$$

The hadronic parameter ξ would be unity in the flavor- $SU(3)$ symmetry limit. Lattice QCD gives[9] $\xi = 1.21^{+0.047}_{-0.035}$, and combining with the experimental result

$$\frac{|V_{td}|}{|V_{ts}|} = 0.2060 \pm 0.0007(\text{exp})^{+0.0081}_{-0.0060}(\text{theory})$$

The error, approximately 3%, is dominated by theory, which comes solely from the error in ξ . There aren't many examples of quantities that the lattice has postdicted (let alone predicted) with this sort of accuracy. So can the rest of us, non-latticists, trust it? On the one hand, because this result is protected by symmetry the required precision is not really 3%. The quantity one must measure is the deviation from the symmetry limit, $\xi^2 - 1$, for which the error is about 25% and perhaps we should be confident that the lattice result is correct at this level. On the other hand, this also tells us that there is a lot of room for improvement. For starters, the determination has been made with only one method (staggered fermions) and it would be reassuring to see the same result from other methods. Also, notice, for comparison, that the leading chiral log calculation[10] gives $\xi \approx 1.15$ with the error in $\xi^2 - 1$ estimated from naive dimensional analysis as $m_K^2/\Lambda_\chi^2 \sim 24\%$, comparable to the

lattice result. So the superiority of the lattice method is not in its current value but in the prospect that it can be improved well beyond the present value. For the lattice to achieve the 0.35% accuracy in ξ needed to match the experimental error in $|V_{td}/V_{ts}|$ a precision of 2% in the determination of $\xi^2 - 1$ is required. Before we, skeptics, trust any significant improvement in this determination, other independent lattice QCD post-dictions of similar accuracy are necessary.

b. $|V_{ub}|$ The magnitude $|V_{ub}|$ determines the rate for $B \rightarrow X_u \ell \nu$. The well known experimental difficulty is that since $|V_{ub}| \ll |V_{cb}|$ the semileptonic decay rate is dominated by charmed final states. To measure a signal it is necessary to either look at exclusive final states or suppress charm kinematically. The interpretation of the measurement requires, in the exclusive case, knowledge of hadronic matrix elements parametrized in terms of form-factors, and for inclusive decays, understanding of the effect of the kinematic cuts on the perturbative expansion and quark-hadron duality.

(i) Inclusive. This has been the method of choice until recently, since it was thought that the perturbative calculation was reliable and systematic and hence could be made sufficiently accurate. However it has become increasingly clear of late that the calculation cannot be made arbitrarily precise. The method uses effective field theories to expand the amplitude systematically in inverse powers of a large energy, either the heavy mass or the energy of the u -quark (or equivalently, of the hadronic final state). One shows that in the restricted kinematic region needed for experiment (to enhance the u -signal to charm-background) the inclusive amplitude is governed by a non-perturbative "shape function," which is, however, universal: it also determines other processes, like the radiative $B \rightarrow X_s \gamma$. So the strategy is to eliminate this unknown, non-perturbative function from the rates for semileptonic and radiative decays.

Surprisingly, most analysis do not eliminate the shape function dependence between the two processes. Instead, practitioners commonly use parametrized fits that unavoidably introduce uncontrolled errors. It is not surprising that errors quoted in the determination of $|V_{ub}|$ are smaller if by a parametrized fit than by the elimination method of [11]. The problem is that parameterized fits introduce errors that are unaccounted for.

Parametrized fits aside, there is an intrinsic problem with the method. Universality is violated by sub-leading terms[12] in the large energy expansion ("sub-leading shape functions"). One can estimate this uncontrolled correction to be of order $\alpha_s \Lambda/m_b$, where Λ is hadronic scale that characterizes the sub-leading effects (in the effective theory language: matrix elements of higher dimension operators). We can try to estimate these effects using models of sub-leading shape functions but then one introduces uncontrolled errors into the determination. At best one should use models to estimate the errors. I think it is fair, albeit unpopular, to say that this method is limited to a precision of about 15%: since there are about

10 sub-leading shape functions, I estimate the precision as $\sqrt{10}\alpha_s\Lambda/m_b$. This is much larger than the error commonly quoted in the determination of $|V_{ub}|$.

This is just as well, since the value of $|V_{ub}|$ from inclusions is in disagreement not only with the value from exclusives but also with the global unitarity triangle fit.

(ii) *Exclusive.* The branching fraction $\mathcal{B}(B \rightarrow \pi\ell\nu)$ is known[13] to 8%. A comparable determination of $|V_{ub}|$ requires knowledge of the $B \rightarrow \pi$ form factor $f_+(q^2)$ to 4%. There are some things we do know about f_+ : (i) The shape is constrained by dispersion relations[14]. This means that if we know f_+ at a few well spaced points we can pretty much determine the whole function f_+ . (ii) We can get a rough measurement of the form factor at $q^2 = m_\pi^2$ from the rate for $B \rightarrow \pi\pi$ [15]. This requires a sophisticated effective theory (SCET) analysis which both shows that the leading order contains a term with $f_+(m_\pi^2)$ and systematically characterizes the corrections to the lowest order SCET. The analysis yields a sensible, but not very accurate, value for $f_+(q^2)$. It is safe to assume that this determination of $f_+(m_\pi^2)$ will not improve beyond the 10% mark.

Lattice QCD can determine the form factor, at least over a limited region of large q^2 . At the moment there is some disagreement between the best two lattice calculations, which however use the same method[16]. A skeptic would require not only agreement between the two existing calculations but also with other methods, not to mention a set of additional independent successful post-dictions, before the result can be trusted for a precision determination of $|V_{ub}|$.

The experimental and lattice measurements can be combined using constraints from dispersion relations and unitarity[17]. Because these constraints follow from fundamentals, they do not introduce additional uncertainties. They improve the determination of $|V_{ub}|$ significantly. The lattice determination is for the q^2 -region where the rate is smallest. This is true even if the form factor is largest there, because in that region the rate is phase space suppressed. But a rough shape of the spectrum is experimentally observed, through a binned measurement[13], and the dispersion relation constraints allows one to combine the full experimental spectrum with the restricted- q^2 lattice measurement. The result of this analysis gives a 13% error in $|V_{ub}|$, completely dominated by the lattice errors.

Alternatives. Exclusive and inclusive determinations of $|V_{ub}|$ have comparable precisions. Neither is very good and the prospect for significant improvement is limited. Other methods need be explored, if not to improve on existing $|V_{ub}|$ to lend confidence to the result. A lattice-free method would be preferable. A third method, proposed a while ago[18], uses the idea of double ratios[19] to reduce hadronic uncertainties. Two independent approximate symmetries protect double ratios from deviations from unity, which are therefore of the order of the product of two small symmetry breaking parameters. For example, the double ra-

tio $(f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d}) = (f_{B_s}/f_{D_s})/(f_{B_d}/f_{D_d}) = 1 + \mathcal{O}(m_s/m_c)$ because $f_{B_s}/f_{B_d} = f_{D_s}/f_{D_d} = 1$ by $SU(3)$ flavor, while $f_{B_s}/f_{D_s} = f_{B_d}/f_{D_d} = \sqrt{m_c/m_b}$ by heavy flavor symmetry. One can extract $|V_{ub}/V_{ts}V_{tb}|$ by measuring the ratio,

$$\frac{d\Gamma(\bar{B}_d \rightarrow \rho\ell\nu)/dq^2}{d\Gamma(\bar{B}_d \rightarrow K^*\ell^+\ell^-)/dq^2} = \frac{|V_{ub}|^2}{|V_{ts}V_{tb}|^2} \cdot \frac{8\pi^2}{\alpha^2} \cdot \frac{1}{N_{\text{eff}}(q^2)} \cdot R_B, \quad (14)$$

where q^2 is the lepton pair invariant mass, and $N_{\text{eff}}(q^2)$ is a computable function. When expressed as functions of the rapidity of the vector meson, $y = E_V/m_V$, the ratios of helicity amplitudes

$$R_B = \frac{\sum_\lambda |H_\lambda^{B \rightarrow \rho}(y)|^2}{\sum_\lambda |H_\lambda^{B \rightarrow K^*}(y)|^2}, \quad R_D = \frac{\sum_\lambda |H_\lambda^{D \rightarrow \rho}(y)|^2}{\sum_\lambda |H_\lambda^{D \rightarrow K^*}(y)|^2},$$

are related by a double ratio: $R_B(y) = R_D(y)(1 + \mathcal{O}(m_s(m_c^{-1} - m_b^{-1})))$. This measurement could be done today: CLEO-c has measured R_D .

A fourth method is available if we are willing to use rarer decays. To extract $|V_{ub}|$ from $\mathcal{B}(B^+ \rightarrow \tau^+\nu_\tau) = (0.88^{+0.68}_{-0.67} \pm 0.11) \times 10^{-4}$ [20] one needs a (lattice?) determination of f_B . Since we want to move away from relying on non-perturbative methods (lattice) to extract V_{ub} we propose a cleaner but more difficult measurement, the double ratio

$$\frac{\Gamma(B_u \rightarrow \tau\nu)}{\Gamma(B_s \rightarrow \ell^+\ell^-)} \sim \frac{|V_{ub}|^2}{|V_{ts}V_{tb}|^2} \cdot \frac{\pi^2}{\alpha^2} \cdot \left(\frac{f_B/f_{B_s}}{f_D/f_{D_s}}\right)^2 \quad (15)$$

In the SM $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) \approx 3.5 \times 10^{-9} \times (f_{B_s}/210 \text{ MeV})^2 (|V_{ts}|/0.040)^2$ is the only presently unknown quantity in the double ratio and is expected to be measured at the LHC.

The ratio $\Gamma(B^+ \rightarrow \tau^+\nu)/\Gamma(B_d \rightarrow \mu^+\mu^-)$ gives us a fifth method. It has basically no hadronic uncertainty, since the hadronic factor $f_B/f_{B_d} = 1$, by isospin. It involves $|V_{ub}|^2/|V_{td}V_{tb}|^2$, an unusual combination of CKMs. In the $\rho - \eta$ plane it forms a circle centered at $\sim (-0.2, 0)$ of radius ~ 0.5 . In a sixth method one studies wrong charm decays $\bar{B}_{d,s} \rightarrow \bar{D}X$ (really $b\bar{q} \rightarrow u\bar{c}$). This can be done both in semi-inclusive decays[21] (an experimentally challenging measurement) or in exclusive decays[22] (where an interesting connection to $B_{d,s}$ mixing matrix elements is involved).

c. $|V_{cb}|$ The method of moments gives a very accurate determination of $|V_{cb}|$ from inclusive semileptonic B decays[23]. In QCD, the rate $d\Gamma(B \rightarrow X_c\ell\nu)/dx dy = |V_{cb}|^2 f(x, y)$, where x and y are the invariant lepton pair mass and energy in units of m_B , is given in terms of four parameters: $|V_{cb}|$, α_s , m_c and m_b . $|V_{cb}|$, which is what we are after, drops out of normalized moments. Since α_s is well known, the idea is to fix m_c and m_b from normalized moments and then use them to compute the normalization, hence determining $|V_{cb}|$. In reality we cannot solve QCD to give the moments in terms of m_c and m_b , but we can use a $1/m_Q$ expansion to write the moments in

terms of m_c , m_b and a few constants that parametrize our ignorance. These constants are in fact matrix elements of operators in the $1/m_Q$ expansion. If terms of order $1/m_Q^3$ are retained in the expansion one needs to introduce five such constants; and additional two are determined by meson masses. All five constants and two quark masses can be over-determined from a few normalized moments that are functions of E_{cut} , the lowest limit of the lepton energy integration. The error in the determination of $|V_{cb}|$ is a remarkably small 2% [24]. But even most remarkable is that this estimate for the error is truly believable. It is obtained by assigning the last term retained in the expansion to the error, as opposed to the less conservative guessing of the first order *not* kept in the expansion. Since there is also a perturbative expansion, the assigned error is of order $\beta_0 \alpha_s^2$, $\alpha_s \Lambda_{\text{QCD}}/m_b$ and $(\Lambda_{\text{QCD}}/m_b)^3$.

As good scientists, let us play Devil's advocate: What, if anything, could go wrong? It seems unlikely that the next order terms could be larger than the terms retained, so the error estimate seems very conservative. There is only one assumption in the calculation that is not fully justified from first principles. The moment integrals can be computed perturbatively (in the $1/m_Q$ expansion) only because the integral can be turned into a contour over a complex E away from the physical region [25]. However, the contour is pinned at the minimal energy, E_{cut} , on the real axis, right on the physical cut. So there is a small region of integration where quark-hadron duality cannot be justified and has to be invoked. How small is the region? Parametrically it is a fraction of order Λ/m_Q , which is a disaster because this is much larger than the claimed error. Nobody really believes this is a problem. For one thing, the fits to moments as functions of E_{cut} are extremely good: the system is over-constrained and the checks are working. And for another, it has been shown [26] that duality works exactly in the SV limit, to order $1/m_Q^2$. But it could very well be that the violation to local quark-hadron duality mainly changes the normalization and has mild dependence on E_{cut} , and that this effect only shows up away from the SV limit.

Fortunately, quark hadron duality can be checked explicitly by considering Lorentzian moments rather than the usual moments (of powers of q^2 or $q_0 = E$). Consider

$$\oint_C dq_0 \frac{L^{\mu\nu} T_{\mu\nu}}{(q_0 - M)^2 + \Delta^2} \quad (16)$$

where L and T are the lepton and hadronic tensors for the semileptonic rate and M and Δ are arbitrary parameters. The contour C consists of segments above and below the cuts on the real axis and closes on a circle at infinity. The integral gets only contributions from the poles at $q_0 = M \pm i\Delta$. If $\Delta > \Lambda_{\text{QCD}}$ one may use of perturbation theory to compute the residues of the poles. This is related to the integral over the discontinuity over the cut. If M is in the interval $(E_{\text{min}}, E_{\text{max}})$ the contribution from the cuts outside the physical region for the semileptonic decay are

power suppressed. So even if these corrections are non-computable, they can be power counted.

The exclusive determination of $|V_{cb}|$ is in pretty good shape theoretically, but is not competitive with the inclusive one. So it provides a sanity check, but not an improvement. The semileptonic rates into either D or D^* are parametrized by functions \mathcal{F} , \mathcal{F}_* , of the rapidity of the charmed meson in the B rest-frame, w . Luke's theorem [27] states $\mathcal{F} = \mathcal{F}_* = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_c)^2$ at $w = 1$. The rate is measured at $w > 1$ and extrapolated to $w = 1$. The extrapolation is made with a first principles calculation to avoid introducing extraneous errors [28]. The result has a 4% error dominated by the uncertainty in the determination of \mathcal{F} , \mathcal{F}_* at $w = 1$.

There is some tension between theory and experiment in these exclusive decays that needs attention. The ratios of form factors $R_{1,2}$ are at variance from theory by three and two sigma respectively [29]. Also, in the heavy quark limit the slopes ρ^2 of \mathcal{F} and \mathcal{F}_* should be equal. One can estimate symmetry violations and obtains [30] $\rho_{\mathcal{F}}^2 - \rho_{\mathcal{F}_*}^2 \simeq 0.19$, while experimentally this is -0.22 ± 0.20 , a deviation in the opposite direction. This is a good place for the lattice to make post-dictions at the few percent error level that may lend it some credibility in other areas where it is needed to determine a fundamental parameter.

V. CONCLUSIONS

From Eq. (12) we learned that we do not need to know V_{ub} (and V_{td}) very well to exclude new flavor physics at the TeV scale. Plugging numbers, (12) gives

$$\Lambda > v \sqrt{\frac{1}{\frac{\delta(\text{CKM})}{\text{CKM}}} \frac{1}{\text{CKM}} \frac{4\pi \sin^2 \theta_w}{\alpha}} \sim 10^3 \text{ TeV} \times \left(\frac{10\%}{\frac{\delta(\text{CKM})}{\text{CKM}}} \right)^{\frac{1}{2}} \left(\frac{0.0002}{\text{CKM}} \right)^{\frac{1}{2}} \quad (17)$$

So 10% precision already makes a strong statement about the scale of new physics, Λ . We argued above that since the solution to the hierarchy problem involves the higgs (or more generally, the breaking of EW symmetry), and since this is responsible for quark/lepton masses, then it is natural that the new physics that solves the hierarchy involves flavor. What gives?

Assuming there are no fine tunings in either the higgs or flavor sectors there must be some symmetry principle that is rendering all of the FCNCs automatically small. The simplest explanation (hence "minimal") is the principle of Minimal Flavor Violation (MFV) (for more details and references see [31]). It can be formulated in the effective field theory language of Sec. III so we do not need to know details of the new TeV-physics. Assuming SM field content below the scale Λ , the SM Lagrangian is supplemented with operators of dimension five and higher, with coefficients of inverse powers of Λ . The MFV princi-

ple automatically gives an additional numerical suppression in the coefficient of these operators, precisely aligned with the CKM factor of the SM. This takes out the factor of $\sqrt{1/\text{CKM}}$ in (17) reducing the right hand side from 10^3 TeV to 10 TeV. I think of this as the modern equivalent of the GIM mechanism, an approximate symmetry of nature that operates even at short distances.

Other, non-minimal, solutions to the problem of the smallness of the coefficients of FCNC operators exist. In the absence of tunings they give at least as large FCNCs as MFV. But they can give effects of the same order as MFV: the trick is to ensure the coefficients are parametrically as small as in MFV. Clearly while these models predict deviations from standard model FCNC's of the same order as MFV, the pattern of deviations is generally different. For more on this see the talk by M. Papucci in these proceedings[32].

I find these arguments compelling. The implications need to be taken seriously. The field of flavor physics should refocus and aim at ruling out deviations from SM FCNCs at the level predicted by MFV (and non-minimal extensions). Moreover, the numerics are rather fortunate. We could have dug our own grave, if MFV predicted that none of the effects of new physics at 10 TeV

could possibly be observable in the near future. But this fortunately is not true. We can well guess where new physics should show up first and where we are barking at the wrong tree.

This effort should proceed regardless of LHC findings: if new physics is indeed found then one should immediately ask if it carries flavor and conforms to the principle of MFV (and extensions). If the new physics is flavor blind a simple explanation is additional new physics at 10^3 TeV, no need for MFV, but for a small hierarchy mechanism (from 1000 TeV down to 10 TeV).

MFV has many surprising implications. But none is more striking than the following. If leptons and quarks unify, and if the solution to the hierarchy problem introduces flavor physics at the TeV scale then lepton flavor violation should be observed in $\mu \rightarrow e$ processes at MEG and PRISM. Exciting flavor physics ahead, indeed!

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